

ESD tr-67-568

67-568



# ESD RECORD COPY

RETURN TO  
SCIENTIFIC & TECHNICAL INFORMATION DIVISION  
(ESTI), BUILDING 1211

## ESD ACCESSION LIST

ESTI Call No.

Copy No.

AL 56679  
Yoff

RADAR CROSS SECTION OF OPEN CIRCULAR LOOPS

Gary A. Thiele

The Ohio State University

## ElectroScience Laboratory

(formerly Antenna Laboratory)

Department of Electrical Engineering  
Columbus, Ohio 43212

TECHNICAL REPORT 2409-3

9 November 1967

Subcontract Number 359

Massachusetts Institute of Technology  
Lincoln Laboratory  
Lexington, Massachusetts 02173

**RECEIVED**

NOV 2 71967

**DISTRIBUTION**

REF ID: A22861

This document has been approved for public release and sale;  
its distribution is unlimited.

REPORT 2409-3

REPORT  
by

The Ohio State University ElectroScience Laboratory  
(Formerly Antenna Laboratory)  
Columbus, Ohio 43212

Purchase Order Number C-363

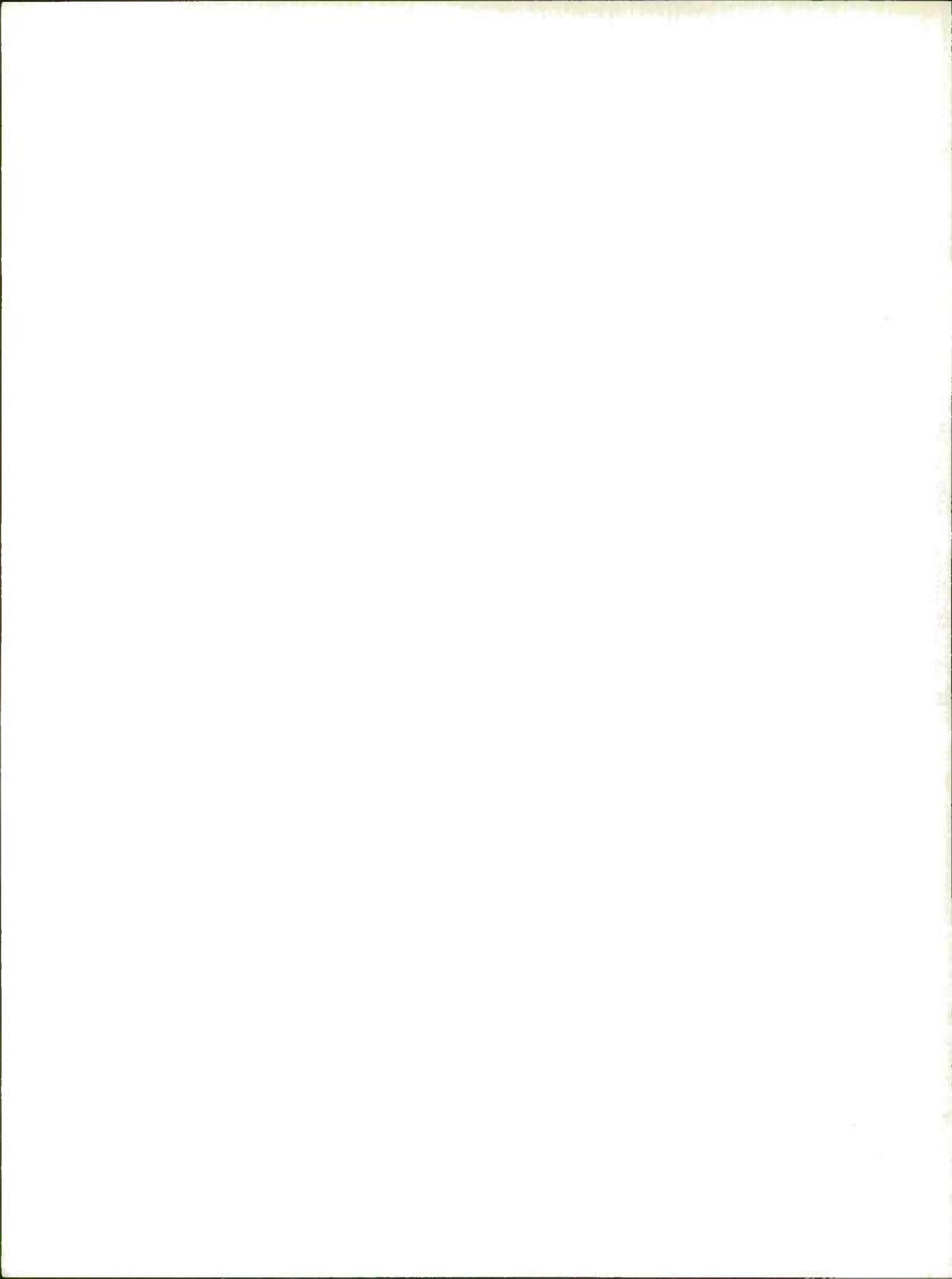
Subcontract Number 359

Prime Contract Number AF 19(628)-5167

## Investigation of Scattering by Thin Wires

**Subject of Report** Radar Cross Section of Open Circular Loops

Date 9 November 1967



## ABSTRACT

Two methods of calculating the monostatic and bistatic scattering by an open circular loop are briefly discussed. Theoretical and experimental backscatter results are presented that show how the echo area varies when a straight wire is curved into an arc of decreasing radius of curvature. For small radii of curvature, an interesting resonance phenomena is observed.

Accepted for the Air Force  
Franklin C. Hudson  
Chief, Lincoln Laboratory Office

## RADAR CROSS SECTION OF AN OPEN CIRCULAR LOOP

Recently, there has been interest in the problem of determining how the radar cross section of a resonant straight wire changes when it is deformed into various shapes. The purpose of this communication is to describe two theoretical approaches to this problem and also to present theoretical and experimental results. Possibly the most important portion of this correspondence is the observance of a narrow band resonance phenomena that occurs when a resonant straight wire is formed into a nearly closed circular loop.

Perhaps the most obvious shape to consider first is that of the open circular loop with radius of curvature  $R$ , and wire radius  $a$ , as shown in Fig. 1. We require that  $R > 10a$  and  $a \ll \lambda$ . The incident plane wave may have any polarization and be incident from any direction.

Two independent theoretical approaches have been applied to this problem. Both utilize the point boundary matching method.<sup>1</sup> The main effort has been directed toward closely approximating the circular arc with  $N$  straight wire segments, each with an unknown uniform current,  $I_n$ . From the vector potential, one can derive a suitable expression for the fields generated by a short hollow tube with a uniform surface current density. These equations are given by Richmond.<sup>2</sup> The tangential electric field from the  $N$  segments is forced to cancel the tangential component of the incident electric field at the center of each segment to generate a system of  $N$  linear equations in the  $N$  unknowns,  $I_n$ . That is,

$$(1) \quad \sum_{n=1}^N C_{mn} I_n = - E_0^i (x_m, z_m)$$

where  $(x_m, z_m)$  is the observation point at the center of segment  $m$ . Solution of the matrix equation yields the currents  $I_n$  from which the echo area may readily be calculated. It has been found that about 80 segments in the case of the open circular loop are needed to obtain good results.

The second approach to the problem has been to consider the arc to be truly circular and represent the current by a Fourier series

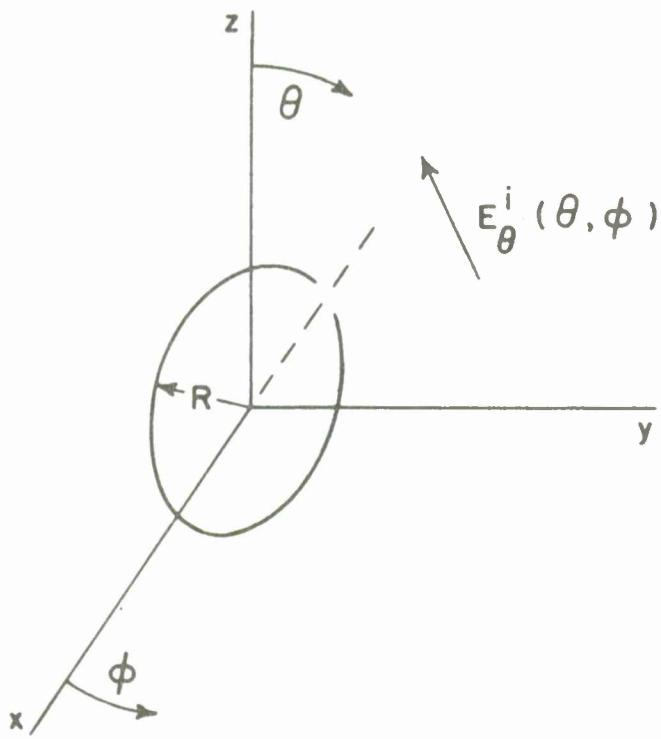


Fig. 1. A  $\theta$ -polarized plane wave is incident on a circular arc in the  $xz$  plane with radius of curvature  $R$ .

$$(2) \quad I = \sum_{n=1}^N I_n \cos (2n - 1) \frac{\pi}{2} \frac{\alpha}{\alpha_0}$$

where  $\alpha$  is defined as the angle from the  $x$ -axis toward the  $z$ -axis and  $\alpha_0$  locates the end of the wire. This series has been chosen such that the current goes to zero at the ends of the wire. From the vector potential, an expression for the field radiated by such a current on an open circular arc may be obtained. By forcing the total tangential electric field to zero at  $N$  points on the radius of the loop, a system of

$N$  equations is generated similar to Eq. (1). Solution of the system of equations yields the coefficients,  $I_n$ , in the Fourier series expansion of the current and calculation of the echo area follows. About twelve terms are needed in the series to obtain good convergence. The computer time for the piecewise-uniform program is generally less than for the Fourier series program, especially for small radii. In addition, our present Fourier series computer program is limited to open circular loops whereas the piecewise-uniform computer program may be used for circular or non-circular, non-planar loops that are either open or closed. Hence, the theoretical results presented here were obtained with the piecewise-uniform method. Nevertheless, the Fourier series program does provide an independent check on the results for circular arcs.

Shown in Fig. 2 are curves of echo area versus the radius of curvature for five different wire lengths. For  $R/\lambda$  equal to 0.15 each wire arc is approximately a semi-circle. Thus, it is apparent that the wire must be curved appreciably beyond a semi-circle before the echo area is grossly effected. For instance, the resonant length for a wire of this radius is  $0.475\lambda$ . When this wire is straight its echo area is about -0.7 dB. However, when  $R/\lambda = 0.15$  it has fallen an additional 1.3 dB to about -2 dB. We further note that as the radius of curvature is decreased the echo area continues to decline until the wire loop becomes almost closed (i. e., the gap between the ends of the wire becomes small). In this region of small  $R/\lambda$  the curves show a distinct resonance phenomena, except for the case  $L = 0.490\lambda$ . A possible explanation for this resonance is the fact that as the gap opening becomes small, the gap capacitance becomes large enough to tune the inductive curved wire and thereby cause a rather sudden increase in the echo area. In fact, one might consider the scatterer to be a loaded circular loop.<sup>3</sup> Further, if the wire is shorter than the resonant length, the resonance is very sharp and does not fall off until the loop is actually shorted or closed as for the  $L = 0.47\lambda$  and  $L = 0.46\lambda$  curves. (The echo area for a closed loop of the circumferences possible in these five cases is on the order of 30 dB below one square wavelength.)<sup>4</sup> However, if the wire is longer than the resonant length then the resonance becomes much less sharp (e. g.,  $L = 0.48\lambda$ ). It may not occur at all if the wire is too long and hence too inductive to be tuned by the small gap capacitance (e. g.,  $L = 0.49\lambda$ ).

To investigate the ability of the theories to predict the existence of such a sensitive resonance phenomena, a few experimental measurements were made. First, five measurements were made at 5.75 GHz using #22 copper wire. The wire length used was  $0.48\lambda$  which is

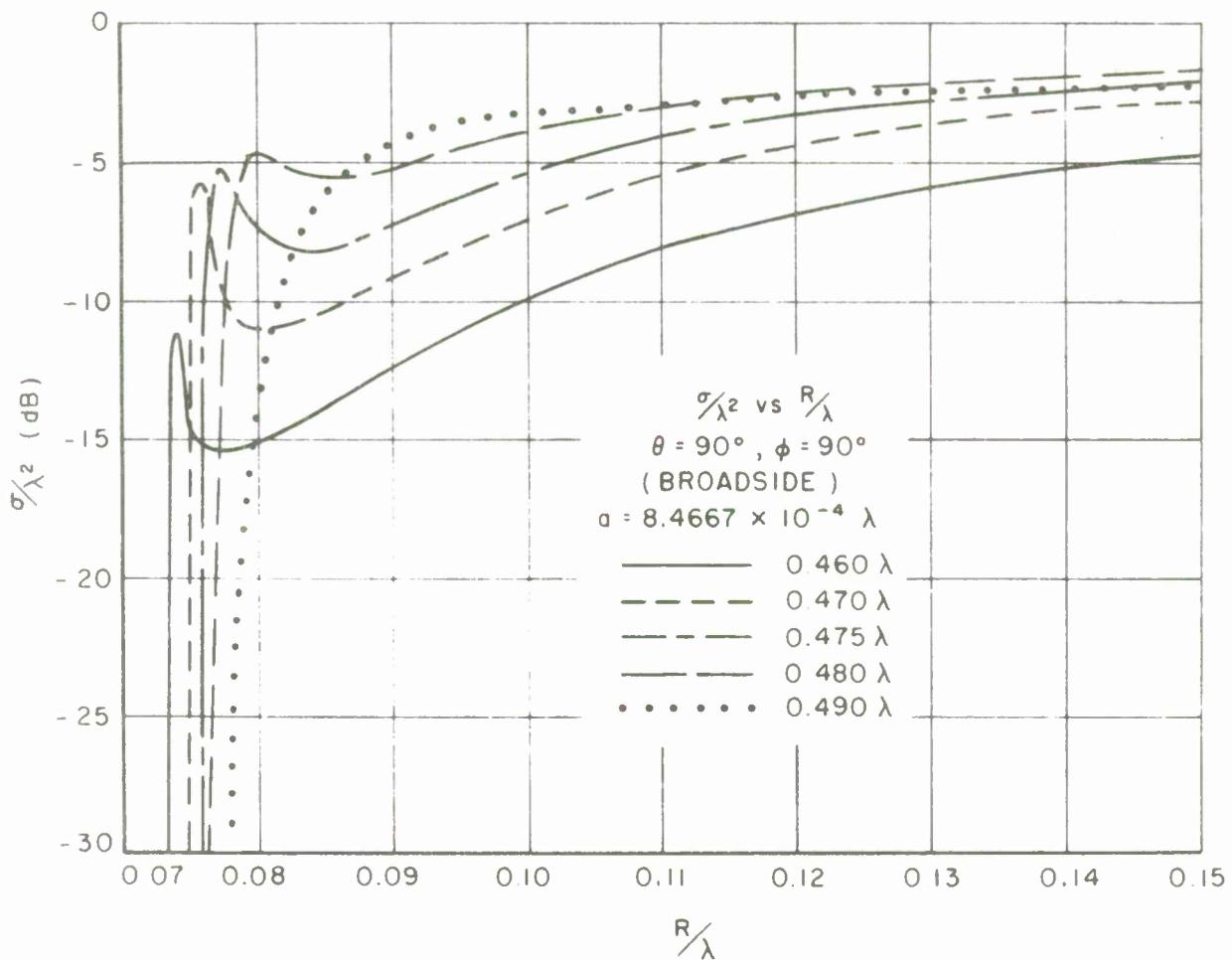
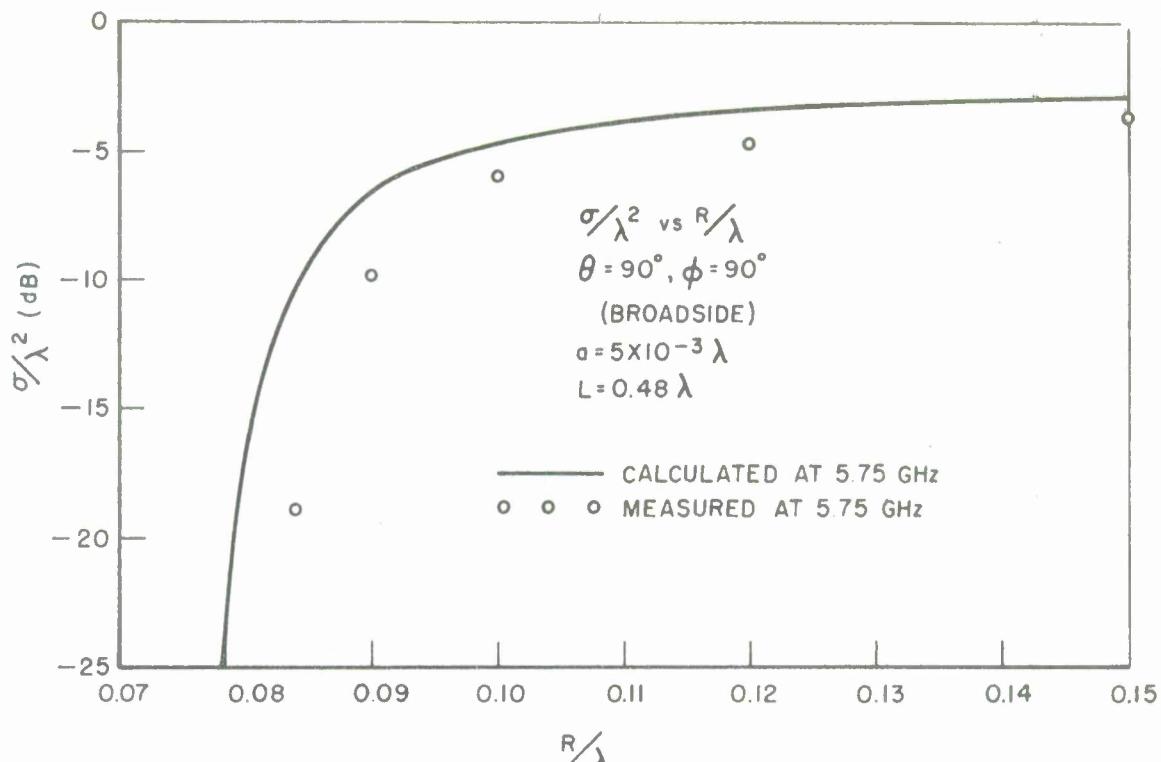
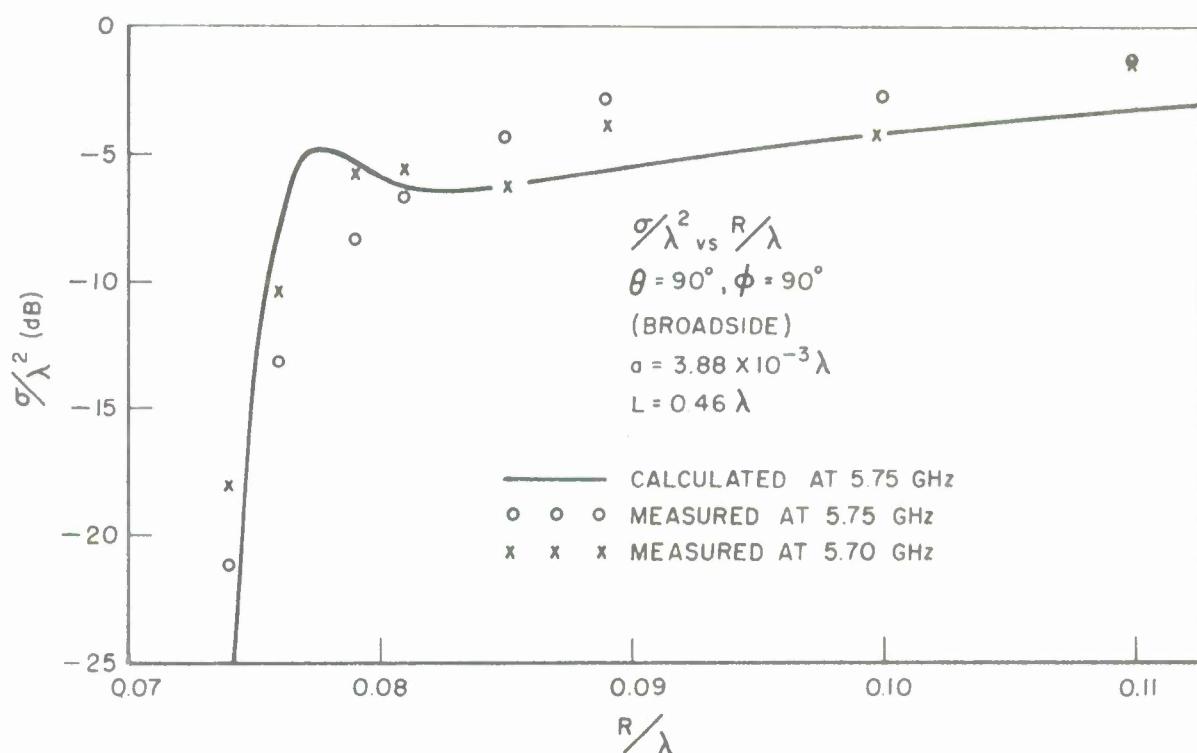


Fig. 2. Backscatter vs. radius of curvature for the broadside case.

longer than the resonant length for a wire of this radius. No resonance was expected and none was observed. These results are shown in Fig. 3a. Next, measurements were made with  $L = 0.46\lambda$  using #26 copper wire at a frequency of 5.75 GHz. Figure 3b shows the theoretical and experimental results obtained for this case. Note that better agreement is obtained if a slightly lower experimental frequency is utilized. This is because there exists a difference between the resonant length of a straight wire when measured experimentally and that predicted in theory (be it point matching or variational). Since the character of the resonance phenomena of interest here is so sensitive to the wire



(a)



(b)

Fig. 3. Experimental results at 5.75 and 5.70 GHz and theoretical results for 5.75 GHz.

length, it is not surprising to see an improvement in Fig. 3b if some allowance is made for this difference between predicted and measured lengths. The measurements shown in Figs. 3a and 3b were made to show a qualitative agreement between theoretical and experimental results.

To obtain truly precision measurements would require extreme accuracy in the construction of the models and perhaps measurement at a much lower frequency where the physical size of the models would be appreciably larger. Nevertheless, the measurements were repeatable and do serve to demonstrate the existence of the gap resonance phenomena on a qualitative basis.

Illustrated in Fig. 4 are two theoretical curves of echo area versus radius of curvature with the gap between the ends of the wire held constant in degrees. An approximate scale for the arc length is

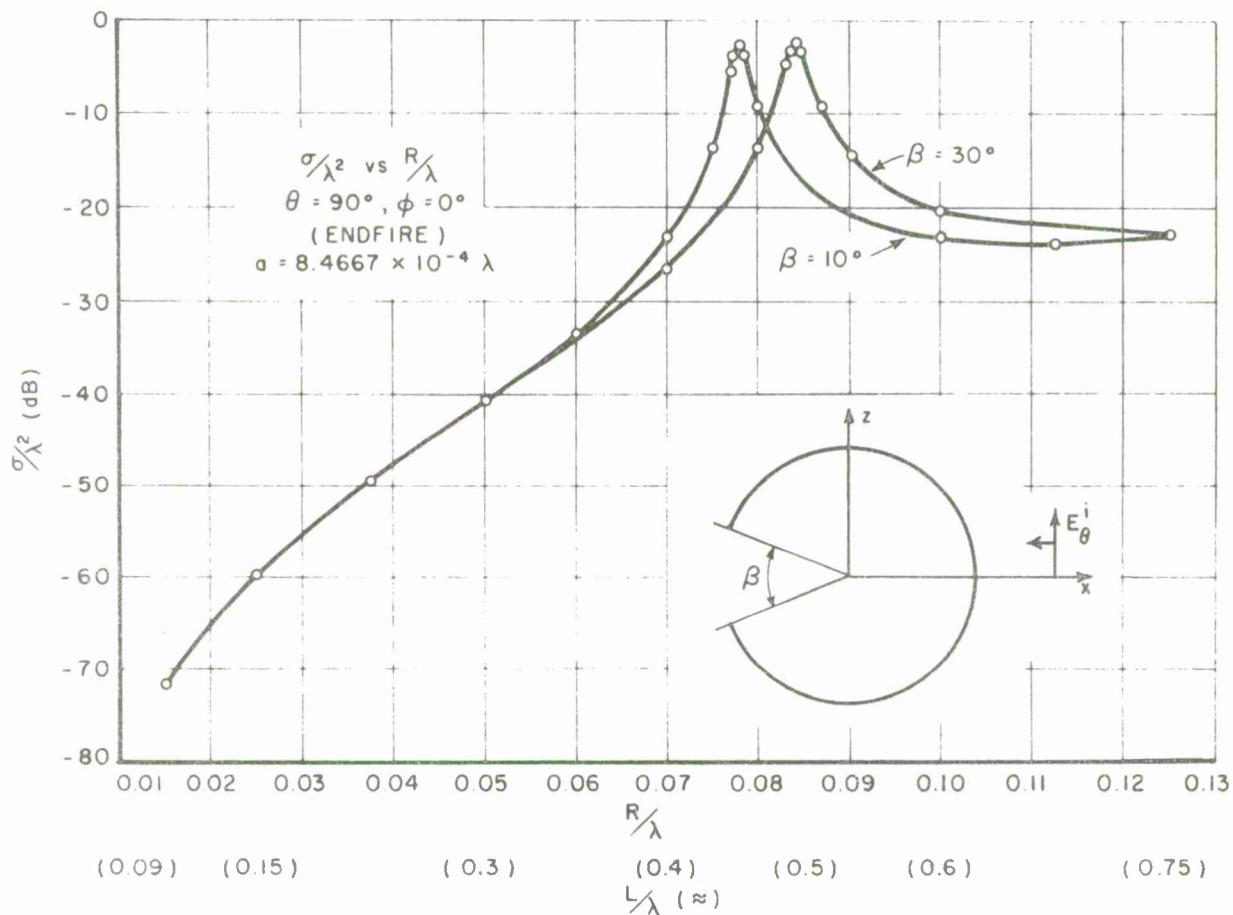


Fig. 4. Backscatter vs. radius of curvature for a constant gap opening in degrees.

indicated in parenthesis. At the low frequency end of the plot the two curves are identical but they are quite different in the vicinity of  $R/\lambda = 0.08$  where  $0.45 < L/\lambda < 0.50$ . In this region the curves exhibit a rather sharp maximum. Specifically, the peak for the  $\beta = 10^\circ$  curve occurs at  $R/\lambda = 0.078$  where  $L/\lambda$  is 0.476 whereas the peak for the  $\beta = 30^\circ$  curve occurs at  $R/\lambda = 0.084$  where  $L/\lambda$  is 0.484.

In conclusion, results have been presented for the scattering by open circular loops. These results show the effect of wire length and radius of curvature on the cross section. In addition, a sensitive narrow-band resonance phenomena was discovered. This phenomena was predicted by both theories and was detected experimentally. It is anticipated that this analysis can be extended to take into account the finite conductivity of the wire by using either a piecewise-uniform or a piecewise-linear representation of the current. Work on the straight wire of perfect conductivity has been similarly extended to one of finite conductivity using a modal expansion of the current.<sup>5</sup>

#### ACKNOWLEDGEMENT

The author is grateful for the assistance of L. M. Schwab in constructing the models and making the measurements. Gratitude is also expressed to Professor J. H. Richmond for his help with the computer program.

## REFERENCES

1. Richmond, J. H., "Digital Computer Solutions of the Rigorous Equations for Scattering Problems," Proc. of the IEEE, Vol. 53, No. 8 (August 1965), pp. 796-804.
2. Richmond, J. H., "A Wire-Grid Model for Scattering by Conducting Bodies," IEEE Transactions on Antennas and Propagation, Vol. AP-14, No. 6 (November 1966), pp. 782-786.
3. Harrington, R. F. and J. L. Ryerson, "Electromagnetic Scattering by Loaded Wire Loops," Radio Science, Vol. 1 (New Series), No. 3 (March 1966), pp. 347-352.
4. Kouyoumjian, R. G., "The Back-Scattering from a Circular Loop (Antenna), Engineering Experiment Station Bulletin No. 162, (November 1956), The Ohio State University, Columbus, Ohio.
5. Thiele, Gary A., "Scattering by a Wire of Finite Conductivity: A Numerical Solution," Report 2169-2, 6 June 1967, Electro-Science Laboratory, The Ohio State University Research Foundation.

**DOCUMENT CONTROL DATA - R&D**

(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)

1. ORIGINATING ACTIVITY (Corporate author) The Ohio State University Research Foundation Under Purchase Order to M.I.T. Lincoln Laboratory		2a. REPORT SECURITY CLASSIFICATION Unclassified
		2b. GROUP None
3. REPORT TITLE  Radar Cross Section of Open Circular Loops		
4. DESCRIPTIVE NOTES (Type of report and inclusive dates)  Technical Report		
5. AUTHOR(S) (Last name, first name, initial)  Thiele, Gary A.		
6. REPORT DATE  9 November 1967		7a. TOTAL NO. OF PAGES  14
8a. CONTRACT OR GRANT NO.  AF 19(628)-5167		7b. NO. OF REFS  5
b. PROJECT NO.  627A		9a. ORIGINATOR'S REPORT NUMBER(S)  2409-3
c.		9b. OTHER REPORT NO(S) (Any other numbers that may be assigned this report)  ESD-TR-67-568
d.		
10. AVAILABILITY/LIMITATION NOTICES  This document has been approved for public release and sale; its distribution is unlimited.		
11. SUPPLEMENTARY NOTES  None		12. SPONSORING MILITARY ACTIVITY  Air Force Systems Command, USAF
13. ABSTRACT  Two methods of calculating the monostatic and bistatic scattering by an open circular loop are briefly discussed. Theoretical and experimental backscatter results are presented that show how the echo area varies when a straight wire is curved into an arc of decreasing radius of curvature. For small radii of curvature, an interesting resonance phenomena is observed.		
14. KEY WORDS  scattering echo area		
curved wires radar cross section		